

Normative Multiagent Systems: A *Dynamic* Generalization

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Abstract

Social norms are powerful formalism in coordinating autonomous agents' behaviour to achieve certain objectives. In this paper, we propose a dynamic normative system to enable the reasoning of the changes of norms under different circumstances, which cannot be done in the existing static normative systems. We study two important problems (norm synthesis and norm recognition) related to the autonomy of the entire system and the agents, and characterise the computational complexities of solving these problems.

1. Introduction

Multiagent systems have been used to model and analyse distributed and heterogeneous systems, with agents being suitable for modelling software processes and physical resources. Roughly speaking, autonomy means that the system by itself, or the agents in the system, can decide for themselves what to do and when to do it [1]. To facilitate autonomous behaviours, agents are provided with capabilities, e.g., to gather information by making observations (via e.g., sensors) and communicating with each other (via e.g., wireless network), to affect the environment and other agents by taking actions, etc. Moreover, systems and agents may have specific objectives to pursue. In this paper, we study autonomy issues related to social norms [2], which are powerful formalism for the coordination of agents, by restricting their behaviour to prevent destructive interactions from taking place, or to facilitate positive interactions [3, 4].

Existing normative systems [5, 6, 7, 4, 8] impose restriction rules on the multiagent systems to disallow agents' actions based on the evaluation of current system state. An implicit assumption behind this setting is that the normative systems do not have (normative) states to describe different social norms under different circumstances. That is, they are *static* normative systems. Specifically, if an action is disallowed on some system state then it will remain disallowed when the same system state occurs again. However, more realistically, social norms may be subject to changes. For example, a human society has different social norms in peacetime and wartime, and an autonomous multiagent system may have different social norms when exposed to different levels of cyber-attacks. This motivates us to propose a new definition of normative systems (in Section 3), to enable the representation of norms under multiple states (hence, *dynamic* normative systems). With a running example, we show that a dynamic normative system can be a necessity if a multiagent system wants to implement certain objectives.

We focus on two related autonomy issues¹. The first is norm synthesis, which is to determine the existence of a normative system for the achievement of objectives. The success of this problem suggests the autonomy of the multiagent system with respect to the objectives, i.e., if all agents in the system choose to conform to the normative system², the objectives can be achieved. For static normative systems, norm synthesis problem is shown to be NP-complete [10]. For our new, and more general, definition of normative systems, we show that it is EXPTIME-complete. This encouraging decidable result shows that the maximum number of normative states can be bounded.

The second is norm recognition, which can be seen as a successive step after deploying an autonomous multiagent systems (e.g., by norm synthesis). For deployed systems such as [11], it can be essential to allow new agents to join anytime. If so, it is generally expected that the new agent is able to recognise the current social norms after playing in the system for a while. Under this general description, we consider two sub-problems related to the autonomy of the system and the new agent, respectively. The

¹In this paper, we consider decision problems of these autonomy issues. The algorithms for the upper bounds in Theorem 1, 2, and 3 can be adapted to implement their related autonomy.

²The synthesised normative system is a common knowledge [9] to the agents.

first one, whose complexity is in PTIME, tests whether the system, under the normative system, can be autonomous in ensuring that the new agent can eventually recognise the norms, no matter how it plays. If such a level of autonomy is unachievable, we may consider the second subproblem, whose success suggests that if the new agent is autonomous (in moving in a smart way) then it can eventually recognise the norms. We show that the second subproblem is PSPACE-complete.

2. Partial Observation Multiagent Systems

A multiagent system consists of a set of agents running in an environment [9]. At each time, every agent takes a local action independently, and the environment updates its state according to agents' joint action. We assume that agents have only partial observations over the system states, because in most real-world systems, agents either do not have the capability of observing all the information (e.g., an autonomous car on the road can only observe those cars in the surrounding area by its sensors or cameras, etc) or are not supposed to observe private information of other agents (e.g., a car cannot observe the destinations of other cars, etc).

Let Agt be a finite set of agents and $Prop$ be a finite set of atomic propositions. A finite multiagent system is a tuple $M = (S, \{Act_i\}_{i \in Agt}, \{L_i\}_{i \in Agt}, \{O_i\}_{i \in Agt}, I, T, \pi)$, where S is a finite set of environment states, Act_i is a finite set of local actions of agent $i \in Agt$ such that $Act = Act_1 \times \dots \times Act_n$ is a set of joint actions, $L_i : S \rightarrow \mathcal{P}(Act_i) \setminus \{\emptyset\}$ provides on every state a nonempty set of local actions that are available to agent i , $I \subseteq S$ is a nonempty set of initial states, $T \subseteq S \times Act \times S$ is a transition relation such that for all $s \in S$ and $a \in Act$ there exists a state s' such that $(s, a, s') \in T$, $O_i : S \rightarrow \mathcal{O}$ is an observation function for each agent $i \in Agt$ such that \mathcal{O} is a set of possible observations, and $\pi : S \rightarrow \mathcal{P}(Prop)$ is an interpretation of the atomic propositions $Prop$ at the states. We require that for all states $s_1, s_2 \in S$ and $i \in Agt$, $O_i(s_1) = O_i(s_2)$ implies $L_i(s_1) = L_i(s_2)$, i.e., an agent can distinguish two states with different sets of next available actions.

Example 1. We consider a business system with two sets of autonomous agents: the producer agents $P = \{p_1, \dots, p_n\}$, and consumer agents $C = \{c_1, \dots, c_m\}$. Let $Agt = P \cup C$.

Each producer agent $p_j \in P$ produces a specific kind of goods with limited quantity each time. There can be more than one agents producing the same goods. We use $g_j \in G$ to denote the kind of goods that are produced by agent p_j , and $b_j \in \mathbb{N}$ to denote the number of goods that can be produced at a time. Every consumer agent $c_i \in C$ has a designated job which needs a set of goods to complete. It is possible that more than one goods of a kind are needed. We use r_i to denote the multiset of goods that are required by agent c_i .

We use $rr_i \subseteq r_i$ to denote the multiset of remaining goods to be collected for c_i , $d_i \in G' = G \cup \{\perp\}$ to represent c_i 's current demand, and $t_i \in P' = P \cup \{\perp\}$ to represent the producer agent from whom c_i is currently requesting goods. Every interaction of agents occurs in two consecutive rounds, and we use $k \in \{1, 2\}$ to denote the current round number.

Because g_j, b_j, r_i do not change their values in a system execution, we assume that they are fixed inputs of the system. The multiagent system M has the state space as

$$S = \{1, 2\} \times \prod_{i \in \{1, \dots, m\}} \{(rr_i, d_i, t_i) \mid rr_i \subseteq r_i, d_i \in G', t_i \in P'\}$$

where the first component $\{1, 2\}$ is for the round number. The initial states are $I = \{1\} \times \prod_{i \in \{1, \dots, m\}} \{(\emptyset, \perp, \perp)\}$.

The consumer agent c_i has a set of actions $Act_{c_i} = \{a_\perp\} \cup \{a_{p_j} \mid p_j \in P\}$. Intuitively, a_\perp means that an agent does nothing, and the action a_{p_j} means that agent c_i sends a request to producer p_j for its goods. The producer agent p_j has a set of actions $Act_{p_j} = \{a_\perp\} \cup \{a_B \mid B \subseteq C, |B| \leq b_j\}$. Intuitively, the action a_B for B a subset of agents represents that agent p_j satisfies the requests from agents in B .

We use pseudocode to describe the transition relation. In the first round, i.e., $k = 1$, it can be described as follows.

RIa. all consumer agents c_i do the following sequential steps:

- (a) if $rr_i = \emptyset$ then we let $rr_i = r_i$. Intuitively, this represents that agent c_i 's job is repeated.
- (b) if $d_i = \perp$ then do the following: let $d_i \in rr_i$, choose an agent p_j such that $d_i = g_j$, and let $t_i = p_j$. Intuitively, if there is no current demand, then

a new demand $d_i \in rr_i$ is generated, and c_i sends a request to a producer agent p_j who is producing goods d_i .

R1b. all producer agents p_j execute action a_\perp , and let $k = 2$.

In the second round, i.e., $k = 2$, it can be described as follows.

R2a. all producer agents p_j do the following sequential steps:

(a) select a maximal subset B of agents such that $B \subseteq \{c_i \mid t_i = p_j\}$ and $|B| \leq b_j$.

Intuitively, from the existing requests, the producer agent p_j selects a set of them according to its production capability.

(b) for all agents c_i in B , let $rr_i = rr_i \setminus \{g_j\}$ and $d_i = t_i = \perp$. Intuitively, if a demand d_i is satisfied, then it is removed from rr_i and we let $d_i = t_i = \perp$.

R2b. all consumer agents execute action a_\perp , and let $k = 1$.

We use “ $\text{var} = \text{val}$ ”, for var a variable and val one of its values, to denote an atomic proposition. Then the labelling function π can be defined naturally over the states. The observation O_i will be discussed in Section 6.

We provide a simple instantiation of the system³. Let $n = 2$, $G = \{g_1, g_2\}$, $b_1 = b_2 = 1$ (two agents produce goods one at each time), $m = 3$, $r_1 = \{g_1\}$, $r_2 = \{g_2\}$ and $r_3 = \{g_1, g_2\}$ (three consumers with the required goods). From the initial state $s_0 = (1, (\emptyset, \perp, \perp), (\emptyset, \perp, \perp), (\emptyset, \perp, \perp))$, we may have the following two states such that $(s_0, (a_\perp, a_\perp, a_{p_1}, a_{p_2}, a_{p_1}), s'_2) \in T$ and $(s'_2, (a_{\{c_1\}}, a_{\{c_2\}}, a_\perp, a_\perp, a_\perp), s'_1) \in T$:

$$\begin{cases} s'_2 = (2, (\{g_1\}, g_1, p_1), (\{g_2\}, g_2, p_2), (\{g_1, g_2\}, g_1, p_1)), \text{ and} \\ s'_1 = (1, (\{\}, \perp, \perp), (\{\}, \perp, \perp), (\{g_1, g_2\}, g_1, p_1)) \end{cases}$$

3. Dynamic Normative Systems

The following is our new definition of normative systems.

³The instantiation is simply to ease the understanding of the definitions in Example 1 and 2. The conclusions for the example system (i.e., Proposition 1, 2, 3, 4, 5) are based on the general definition.

Definition 1. A dynamic normative system of a multiagent system

$M = (S, \{Act_i\}_{i \in Agt}, \{L_i\}_{i \in Agt}, \{O_i\}_{i \in Agt}, I, T, \pi)$ is a tuple $N_M = (Q, \delta_n, \delta_u, q_0)$ such that Q is a set of normative states, $\delta_n : S \times Q \rightarrow \mathcal{P}(Act)$ is a function specifying, for each environment state and each normative state, a set of joint actions that are disallowed, $\delta_u : Q \times S \rightarrow Q$ is a function specifying the update of normative states according to the changes of environment states, and q_0 is the initial normative state.

A (static) normative system in the literature can be seen as a special case of our definition where the only normative state is q_0 . In such case, we have $Q = \{q_0\}$, $\delta_u(q_0, s) = q_0$ for all $s \in S$, and can therefore write the function δ_n as function $\delta : S \rightarrow \mathcal{P}(Act)$. It is required that the function δ_n (and thus δ) does not completely eliminate agents' joint actions, i.e., $\delta_n(s, q) \subset \Pi_{i \in Agt} L_i(s)$ for all $s \in S$ and $q \in Q$.

We give two dynamic normative systems.

Example 2. Let M be the multiagent system given in Example 1. Let s_1 and s_2 range over those environmental states such that $k = 1$ and $k = 2$, respectively.

The normative system $N_M^1 = (Q^1, \delta_n^1, \delta_u^1, q_0^1)$ is such that:

- $Q^1 = \Pi_{p_j \in P} \{1, \dots, m\}$, where each producer maintains a number indicating the consumer whose requirement must be satisfied in this normative state,
- $\delta_n^1(s_1, q) = \emptyset$, i.e., no joint actions are disallowed on s_1 , and $(a_{B_1}, \dots, a_{B_n}, a_\perp, \dots, a_\perp) \in \delta_n^1(s_2, (y_1, \dots, y_n))$ if there exists $j \in \{1, \dots, n\}$ such that $B_j \subseteq C$ and $c_{y_j} \notin B_j$. Intuitively, for producer agent p_j , an action a_{B_j} is disallowed on the second round if B_j does not contain the consumer c_{y_j} who is needed to be satisfied in this round.
- $\delta_u^1(q, s_2) = q$ and $\delta_u^1((y_1, \dots, y_n), s_1) = (y'_1, \dots, y'_n)$ such that $y'_j = (y_j \bmod m) + 1$ for $j \in \{1, \dots, n\}$; intuitively, the normative state increments by 1 and loops forever.
- $q_0^1 = (1, \dots, n)$, i.e., producer agents p_j start from c_j .

For the instantiation in Example 1, we have that

- $Q^1 = \{1, 2, 3\} \times \{1, 2, 3\}$, $q_0^1 = (1, 2)$,

- $(a_{B_1}, a_{B_2}, a_{\perp}, a_{\perp}, a_{\perp}) \in \delta_n^1(s'_2, (1, 2))$ if either $B_1 \in \{\emptyset, \{c_2\}, \{c_3\}, \{c_2, c_3\}\}$ or $B_2 \in \{\emptyset, \{c_1\}, \{c_3\}, \{c_1, c_3\}\}$,
- $\delta_u^1((1, 2), s_1) = (2, 3), \delta_u^1((2, 3), s_1) = (3, 1)$.

We define another normative system $N_M^2 = (Q^2, \delta_n^2, \delta_u^2, q_0^2)$ by extending the number maintained by each producer into a first-in-first-out queue so that the ordering between consumers who have sent the requests matters. That is, we have $Q^2 = \Pi_{p_j \in P}(\{\epsilon\} \cup \{i_1 \dots i_k \mid k \in \{1, \dots, m\}, i_x \in C \text{ for } 1 \leq x \leq k\})$ where the symbol ϵ denotes an empty queue, and $q_0^2 = \Pi_{p_j \in P}\{\epsilon\}$ which means that producers start from empty queues. The functions δ_n^2 and δ_u^2 can be adapted from N_M^1 , and details are omitted here.

The following captures the result of applying a normative system on a multiagent system, which is essentially a product of these two systems.

Definition 2. Let M be a multiagent system and N_M a normative system on M , the result of applying N_M on M is a Kripke structure $K(N_M) = (S^\dagger, I^\dagger, T^\dagger, \pi^\dagger)$ such that

- $S^\dagger = S \times Q$ is a set of states,
- $I^\dagger = I \times \{q_0\}$ is a set of initial states,
- $T^\dagger \subseteq S^\dagger \times S^\dagger$ is such that, for any two states (s_1, q_1) and (s_2, q_2) , we have $((s_1, q_1), (s_2, q_2)) \in T^\dagger$ if and only if, (1) there exists an action $a \in \text{Act}$ such that $(s_1, a, s_2) \in T$ and $a \notin \delta_n(s_1, q_1)$, and (2) $q_2 = \delta_u(q_1, s_2)$. Intuitively, the first condition specifies the enabling condition to transit from state s_1 to state s_2 by taking a joint action a which is allowed in the normative state q_1 . The second condition specifies that the transition relation needs to be consistent with the changes of normative states.
- $\pi^\dagger : S^\dagger \rightarrow \mathcal{P}(\text{Prop})$ is such that $\pi^\dagger((s, q)) = \pi(s)$.

Example 3. For the instantiation, in the structure $K(N_M^1)$, we have

$((s_0, (1, 2)), (s'_2, (1, 2))), ((s'_2, (1, 2)), (s'_1, (2, 3))) \in T^\dagger$. The latter is because $(s'_2, (a_{\{c_1\}}, a_{\{c_2\}}, a_{\perp}, a_{\perp}, a_{\perp}), s'_1) \in T$, $\{c_1\} \notin \{\emptyset, \{c_2\}, \{c_3\}, \{c_2, c_3\}\}$, and $\{c_2\} \notin \{\emptyset, \{c_1\}, \{c_3\}, \{c_1, c_3\}\}$.

For $a = (a_{\{c_1\}}, a_{\{c_3\}}, a_{\perp}, a_{\perp}, a_{\perp})$ and $s_1'' = (1, (\{\}, \perp, \perp), (\{g_2\}, g_2, p_2), (\{g_1\}, \perp, \perp))$, we have $(s_2', a, s_1'') \in T$ but $((s_2', (1, 2)), (s_1'', (2, 3))) \notin T^\dagger$. This is because, for p_2 , it is required to make c_2 as its current priority according to the normative state, and cannot choose to satisfy c_3 instead.

We remark that, the normative system, as many current formalisms, imposes hard constraints on the agents' behaviour. As stated in e.g., [12], social norms may be soft constraints that agents can choose to comply with or not. To accommodate soft social norms, we can redefine the function δ_n as $\delta_n : S \times Q \times Act \rightarrow U$ to assign each joint action a cost utility for every agent, on each environment state and normative state. With this definition, norms become soft constraints: agents can choose to take destructive actions, but are encouraged to avoid them due to their high costs. The objective language to be introduced in the next section also needs to be upgraded accordingly to express properties related to the utilities. We leave such an extension as a future work.

4. Objective Language

To specify agents' and the system's objectives, we use temporal logic CTL [13] whose syntax is as follows.

$$\phi ::= p \mid \neg\phi \mid \phi_1 \vee \phi_2 \mid EX\phi \mid E(\phi_1 U \phi_2) \mid EG\phi$$

where $p \in Prop$. Intuitively, formula $EX\phi$ expresses that ϕ holds at some next state, $E(\phi_1 U \phi_2)$ expresses that on some path from current state, ϕ_1 holds until ϕ_2 becomes true, and $EG\phi$ expresses that on some path from current state, ϕ always holds. Other operators can be obtained as usual, e.g., $EF\phi \equiv E(True U \phi)$, $AG\phi \equiv \neg E(True U \neg\phi)$, $AF\phi = \neg EG\neg\phi$ etc.

A path in a Kripke structure $K(N_M)$ is a sequence $s_0 s_1 \dots$ of states such that $(s_i, s_{i+1}) \in T^\dagger$ for all $i \geq 0$. The semantics of the language is given by a relation $K(N_M), s \models \phi$ for $s \in S^\dagger$, which is defined inductively as follows [13]:

1. $K(N_M), s \models p$ if $p \in \pi^\dagger(s)$,
2. $K(N_M), s \models \neg\phi$ if not $K(N_M), s \models \phi$,

3. $K(N_M), s \models \phi_1 \vee \phi_2$ if $K(N_M), s \models \phi_1$ or $K(N_M), s \models \phi_2$,
4. $K(N_M), s \models EX\phi$ if there exists a state $s' \in S^\dagger$ such that $(s, s') \in T^\dagger$ and $K(N_M), s' \models \phi$,
5. $K(N_M), s \models E(\phi_1 U \phi_2)$ if there exists a path $s_0 s_1 \dots$ and a number $n \geq 0$ such that $s_0 = s$, $K(N_M), s_k \models \phi_1$ for $0 \leq k \leq n-1$ and $K(N_M), s_n \models \phi_2$,
6. $K(N_M), s \models EG\phi$ if there exists a path $s_0 s_1 \dots$ such that $s_0 = s$ and $K(N_M), s_k \models \phi$ for all $k \geq 0$.

The verification problem, denoted as $K(N_M) \models \phi$, is, given a multiagent system M , its associated normative system N_M , and an objective formula ϕ , to decide whether $K(N_M), s \models \phi$ for all $s \in I^\dagger$. The norm synthesis problem is, given a system M and an objective formula ϕ , to decide the existence of a normative system N_M such that $K(N_M) \models \phi$. The norm recognition problem will be defined in Section 6. For the measurement of the complexity, we take the standard assumption that the sizes of the multiagent system and the normative system are measured with the number of states, and the size of the objective formula is measured with the number of operators.

Example 4. *For the system in Example 1, interesting objectives expressed in CTL may include*

$$\phi_1 \equiv \bigwedge_{i \in C} \bigwedge_{j \in P} AG (t_i = p_j \Rightarrow EF d_i = \perp)$$

which says that it is always the case that if there is a request from a consumer c_i to a producer p_j (i.e., $t_i = p_j$), then the request is possible to be satisfied eventually (i.e., $d_i = \perp$), and

$$\phi_2 \equiv \bigwedge_{i \in C} \bigwedge_{j \in P} AG (t_i = p_j \Rightarrow AF d_i = \perp)$$

which says that it is always the case that if there is a request from a consumer c_i to a producer p_j , then on all the paths the request will eventually be satisfied. Both ϕ_1 and ϕ_2 are liveness objectives that are important for an ecosystem to guarantee that no agent can be starving forever. The objective ϕ_2 is stronger than ϕ_1 , and their usefulness is application-dependent. The following proposition shows that static normative systems are insufficient to guarantee the satisfiability of the objectives in this ecosystem.

Proposition 1. *There exists an instance of a multiagent system M such that, for all static normative systems N_M , we have that $K(N_M) \not\models \phi_1 \wedge \phi_2$.*

The proof idea is based on the following simple case. Assume that there are one producer p_1 , such that $b_1 = 1$, and two consumers c_1 and c_2 , such that $r_1 = r_2 = \{g_1\}$. There only exist the following three static normative systems which have different restrictions on an environment state $s_2 = (2, (\{g_1\}, g_1, p_1), (\{g_1\}, g_1, p_1))$: (Recall that q_0 is the only normative state in static normative systems.)

- N_M^3 is such that $\delta_n^3(s_2, q_0) = \{a_{\{c_1\}}\}$, i.e., c_1 is not satisfied.
- N_M^4 is such that $\delta_n^4(s_2, q_0) = \{a_{\{c_2\}}\}$, i.e., c_2 is not satisfied.
- N_M^5 is such that $\delta_n^5(s_2, q_0) = \emptyset$, i.e., no restriction is imposed.

We can see that $K(N_M^h) \not\models \phi_1$ for $h \in \{3, 4\}$ and $K(N_M^5) \not\models \phi_2$. The former is because one of the agents is constantly excluded from being satisfied. For the latter, there exists an infinite path $s_0(s_2s_1)^\infty$ such that $s_0 = (1, (\emptyset, \perp, \perp), (\emptyset, \perp, \perp))$ is an initial state, s_2 is given as above, and $s_1 = (1, (\emptyset, \perp, \perp), (\{g_1\}, g_1, p_1))$ is the state on which consumer c_1^1 's requirement is satisfied. On this path, the requirement from c_1^2 is never satisfied. On the other hand, for the dynamic normative systems in Example 2, all the consumers' requests can be satisfied, so we have the following conclusion.

Proposition 2. *Given a system M and a normative system N_M^1 or N_M^2 , we have that $K(N_M^h) \models \phi_1 \wedge \phi_2$ for $h \in \{1, 2\}$.*

The above example suggests that, to achieve some objectives, we need *dynamic* normative systems to represent the changes of social norms under different circumstances. Then, another question may follow about the maximum number of normative states. The dynamic system could be uninteresting if the number of states can be infinite. Fortunately, in the next section, we show with the complexity result that, for objectives expressed with CTL formulas, in the worst case, an exponential number of normative states are needed.

5. The Complexity of Norm Synthesis

We have the following result for norm synthesis.

Theorem 1. *The norm synthesis problem is EXPTIME-complete, with respect to the sizes of the system and the objective formula.*

Proof: We first show the upper bound: **EXPTIME Membership**.

From $M = (S, \{Act_i\}_{i \in Agt}, \{L_i\}_{i \in Agt}, \{O_i\}_{i \in Agt}, I, T, \pi)$, we define a Büchi tree automaton $A_M = (\Sigma, D, Q, \delta, q_0, Q)$ such that

1. $\Sigma = \mathcal{P}(V) \cup \{\perp\}$, $Q = S \times \{\top, \vdash, \perp\}$, $q_0 = (s, \top)$ for $s \in I$,
2. $D = \bigcup_{s \in S} \{1, \dots, |T(s)|\}$ where $T(s) = \{s' \mid \exists a \in Act : (s, a, s') \in T\}$,
3. $\delta : Q \times \Sigma \times D \rightarrow 2^{Q^*}$ is defined as follows: for $s \in S$ and $k = |T(s)|$ with $T(s) = (s_1, \dots, s_k)$, we have (a) if $m \in \{\vdash, \perp\}$ then $\delta((s, m), \perp, k) = \{((s_1, \perp), \dots, (s_k, \perp))\}$, and (b) if $m \in \{\vdash, \top\}$, then we let $((s_1, y_1), \dots, (s_k, y_k)) \in \delta((s, m), \pi(s), k)$ such that, there exists a nonempty set $B \subseteq \{1, \dots, k\}$ of indices such that
 - (a) $y_i = \top$, for all $i \in B$, and
 - (b) $y_j = \vdash$, for all $j \notin B$ and $1 \leq j \leq k$.

Note that we use $\mathcal{F} = Q$ to express that we only care about infinite paths. Moreover, the formula ϕ needs to be modified to reject those runs where \perp is labeled on the states. This can be done by following the approach in [14]. We still call the resulting formula ϕ .

Given a CTL formula ϕ and a set $D \subset \mathbb{N}$ with a maximal element k , there exists a Büchi tree automaton $A_{D, \neg\phi}$ that accepts exactly all the tree models of $\neg\phi$ with branching degrees in D . By [15], the size of $A_{D, \neg\phi}$ is $O(2^{k \cdot |\phi|})$. The norm synthesis problem over M and ϕ for ϕ a CTL formula is equivalent to checking the emptiness of the product automaton $A_M \times A_{D, \neg\phi}$. The checking of emptiness of Büchi tree automaton can be done in quadratic time, so the norm synthesis problem for ϕ a CTL formula can be done in exponential time.

Therefore, the norm synthesis problem over M and ϕ can be done in exponential time with respect to $|S|$, $|\bigcup_{i \in \text{Agt}} \text{Acts}_i|$, and $|\phi|$. That is, it is in EXPTIME.

We then show the lower bound: **EXPTIME Hardness**

The lower bound is reduced from the problem of a linearly bounded alternating Turing machine (LBATM) accepting an empty input tape, which is known to be EXPTIME-complete [16]. Let AT be an LBATM. A system $M(AT)$ of a single agent is constructed such that the agent moves on \exists states and the environment moves on \forall states. The normative system, applied on the agent's behaviour, may prune some branches of the system. We use an objective formula ϕ to express that the resulting system correctly implements several modification rules (which makes the resulting system moves as the AT does) and all paths lead to accepting states. Therefore, the norm synthesis problem on the system $M(AT)$ and the objective formula ϕ is equivalent to the acceptance of the automaton AT on empty tape. That is, the complexity is EXPTIME hard.

An alternating Turing machine AT is a tuple $(Q, \Gamma, \delta, q_0, g)$ where Q is a finite set of states, Γ is a finite set of tape symbols including a blank symbol \perp , $\delta : Q \times \Gamma \rightarrow \mathcal{P}(Q \times \Gamma \times \{-1, +1\})$ is a transition function, $q_0 \in Q$ is an initial state, $g : Q \rightarrow \{\forall, \exists, \text{accept}, \text{reject}\}$ specifies the type of each state. We use $b \in \Gamma$ to denote the blank symbol. The input w to the machine is written on the tape. We use w_i to denote the alphabet written on the i th cell of the tape.

The size of a Turing machine is defined to be the size of space needed to record its transition relation, i.e., $2 \times |\Gamma|^2 \times |Q|^2$. An LBATM is an ATM which uses m tape cells for a Turing machine description of size m . Let $L = \{1, \dots, m\}$. A configuration of the machine contains a state $q \in Q$, the header position $h \in L$, and the tape content $v \in \Gamma^*$. A configuration $c = (q, h, v)$ is accepting if $g(q) = \text{accept}$, or $g(q) = \forall$ and all successor configurations are accepting, or $g(q) = \exists$ and at least one of the successor configuration is accepting. The machine AT accepts an empty tape if the initial configuration of M (the state of M is q_0 , the head is at the left end of the tape, and all tape cells contain symbol \perp) is accepting, and to reject if the initial configuration is rejecting. It is known that the problem is EXPTIME-complete.

We construct a multiagent system M with a single agent i . We have

$M = (S, \{Act_i\}_{i \in Agt}, \{L_i\}_{i \in Agt}, \{O_i\}_{i \in Agt}, I, T, \pi)$ where

1. $S = (Q \times L) \cup (Q \times L \times \Gamma) \cup (Q \times L \times L \times \Gamma)$,
2. $Act_i = \{a_2, a_3\} \cup \{a_{rcd} \mid r \in Q, c \in \Gamma, d \in \{-1, +1\}\} \cup \Sigma$,
3. the function L_i is defined as
 - (a) $L_i((t_i, o_{q,h})) = \Sigma$,
 - (b) $L_i((t_i, o_{q,h,b})) = \{a_2\}$ for $g(q) = \forall$,
 - (c) $L_i((t_i, o_{q,h,b})) = \{a_{rcd} \mid r \in Q, c \in \Gamma, d \in \{-1, +1\}\}$ for $(r, c, d) \in \delta(q, b)$ and $g(q) = \exists$, and
 - (d) $L_i((t_i, o_{q,h_1,h_2,c})) = \{a_3\}$.
4. the function O_i is defined as follows: $O_i((q, h)) = o_{q,h}$, $O_i((q, h, b)) = o_{q,h,b}$,
 $O_i((q, h_1, h_2, c)) = o_{q,h_1,h_2,c}$.
5. $I = \{(q_0, 1)\}$,
6. the transition relation T is defined as follows:
 - (a) $((q, h), b, (q, h, b)) \in T$ for $b \in \Gamma$.
 - (b) $((q, h, b), a_2, (r, h + d, h, c)) \in T$ for $(r, c, d) \in \delta(q, b)$ and $g(q) = \forall$.
 - (c) $((q, h, b), a_{rcd}, (r, h + d, h, c)) \in T$ for $g(q) = \exists$.
 - (d) $((r, h_1, h_2, c), a_3, (r, h_1)) \in T$.

Intuitively, a transition $(r, c, d) \in \delta(q, b)$ is simulated by three consecutive transitions:

1. $((q, h), b, (q, h, b))$, where the agent guesses the correct symbol written in cell h .
2. $((q, h, b), a, (r, h + d, h, c))$ such that if $a = a_2$ then the state q is an \forall state and it is the environment that moves according to δ , and if $a = a_{rcd}$ then the state q is an \exists state and it is the agent that moves according to δ .
3. $((r, h + d, h, c), a_3, (r, h_1))$, where the system makes a deterministic transition.

For the second transition, we let the environment move on \forall states because all the successor states have to be explored, while let the agent move on \exists states so that it allows the normative system to prune some branches.

Let $V = \{Q_q \mid q \in Q\} \cup \{H_h, G_h \mid h \in L\} \cup \{R_b, W_b \mid b \in \Gamma\} \cup \{acc, k_2\}$ be a set of boolean variables. Intuitively, Q_q represents that the current state is q , H_h represents that the current header position is h , G_h represents that the last header position is h , R_b represents that the symbol on the current cell is b , and W_b represents that the symbol written on the last header position is b . We define the labelling function π as follows: $\{Q_q, H_h\} \subseteq \pi((q, h))$, $\{Q_q, H_h, R_b, k_2\} \subseteq \pi((q, h, b))$, $\{Q_r, H_{h_1}, G_{h_2}, W_c\} \subseteq \pi((r, h_1, h_2, c))$, $acc \in \pi((q, h))$ if $g(q) = accept$. Moreover, we let $\mathcal{F} = \emptyset$.

We need the following formulas.

1. Formula $\phi_1(h) \equiv A((H_h \wedge k_2 \Rightarrow R_\perp) \ U \ \bigvee_{c \in \Gamma} (G_h \wedge W_c))$ expresses that the symbol on position h is \perp until it is modified.
2. Formula $\phi_2(h, b) \equiv G_h \wedge W_b \Rightarrow A((H_h \wedge k_2 \Rightarrow R_b) \ U \ \bigvee_{c \in \Gamma} (G_h \wedge W_c))$ expresses that once the symbol on position h is modified into b , it will stay the same until the next modification occurs.

Then the formula to be model checked on the system is

$$\phi = (\bigwedge_{h \in L} \phi_1(h) \wedge AG \bigwedge_{h \in L} \bigwedge_{b \in \Gamma} \phi_2(h, b)) \wedge AFacc$$

Intuitively, the norm synthesis problem over M and ϕ is to determine the existence of a normative system, which by pruning the behaviour of the agent, can make the resulting system correctly implements the modification rules and all branches can be accepting. Therefore, the norm synthesis problem is equivalent to the acceptance of the automaton AT on empty tape. That is, the complexity is EXPTIME hard. \square

6. Agent Recognition of Social Norms

For a multiagent system to be autonomous without human intervention, it is important that it can maintain its functionality when new agents join or old agents leave. For

a new agent to join and function well, it is essential that it is capable of recognising the social norms that are currently active. As stated in the previous sections, the agent has only partial observation over the system state, and is not supposed to observe the social norms. On the other hand, it is also unrealistic to assume that the agent does not know anything about the social norms of the system it is about to join. Agent is designed to have a set of prescribed capabilities and is usually supposed to work within some specific scenarios. Therefore, the actual situation can be that, the agent knows in prior that there are a set of possible normative systems, one of which is currently applied on the multiagent system. We remark that, assuming a set of normative systems does not weaken the generality of the setting, because Theorem 1 implies that there are a finite number of possible normative systems (subject to a bisimulation relation between Kripke structures). This situation naturally leads to the following two new problems:

- **(NC₁)** to determine whether the agent can always recognise which normative system is currently applied; and
- **(NC₂)** to determine whether the agent can find a way to recognise which normative system is currently applied.

The successful answer to the problem NC_1 implies the successful answer to the problem NC_2 , but not vice versa. Intuitively, the successful answer of NC_1 implies a high-level autonomy of the system that the new agent can be eventually incorporated into the system no matter how it behaves. We assume that once learned the social norms the new agent will behave accordingly. If such an autonomy of the system cannot be achieved, the successful answer of NC_2 implies a high-level autonomy of the agent that, by moving in a smart way, it can eventually recognise the social norms.

We formalise the problems first. Let Ψ be a set of possible normative systems defined on a multiagent system, $Path(K(N))$ be the set of possible paths of the Kripke structure $K(N)$ for $N \in \Psi$. We assign every normative system in Ψ a distinct index, denoted as $ind(N)$. This index is attached to every path $\rho \in Path(K(N))$, and let $ind(\rho) = ind(N)$.

Let the new agent be x such that $x \notin Agt$ and its observation function be O_x . For any state $(s, q) \in S^\dagger$, we define a projection function $\widehat{(s, q)} = s$. So $\widehat{\rho}$ is the projection of

a path ρ of a Kripke structure to the associated multiagent system. We extend O_x to the paths of Kripke structure $K(N)$ as follows: $O_x(\rho s^\dagger) = O_x(\rho) \cdot O_x(\widehat{s^\dagger})$ for $\rho \in \text{Path}(K(N))$ and $s^\dagger \in S^\dagger$. We have $O_x(\epsilon) = \epsilon$, which means when a path is empty, the observation is also empty. We also define its inverse O_x^{-1} which gives a sequence o of observations, returns a set of possible paths ρ on which agent x 's observations are o , i.e.,

$$O_x^{-1}(o) = \{\rho \in \text{Path}(K(N)) \mid O_x(\rho) = o, N \in \Psi\}.$$

W.l.o.g., we assume that $N_0 \in \Psi$ is the active normative system. Let \mathbb{N} be the set of natural numbers, we have

Definition 3. *NC_1 problem is the existence of a number $k \in \mathbb{N}$ such that for all paths $\rho \in \text{Path}(K(N_0))$ such that $|\rho| \geq k$, we have that $\rho' \in O_x^{-1}(O_x(\rho))$ implies that $\text{ind}(\rho') = \text{ind}(\rho)$.*

NC_2 problem is the existence of a path $\rho \in \text{Path}(K(N_0))$ such that for all $\rho' \in O_x^{-1}(O_x(\rho))$ we have $\text{ind}(\rho') = \text{ind}(\rho)$.

Intuitively, NC_1 states that as long as the path is long enough, the new agent can eventually know that the active normative system is N_0 . That is, no matter how the new agent behaves, it can eventually recognise the current normative system. On the other hand, NC_2 states that such a path exists (but not necessarily for all paths). That is, to recognise the normative system, the new agent needs to move smartly.

Example 5. *For the system in Example 1, we assume a new consumer agent c_v such that $v = m + 1$. Also, we define $O_{c_v}(s) = \{c_i \mid c_i \in C, t_i(s) = t_v(s) \neq \perp\}$ for all $s \in S$. Intuitively, the agent c_v keeps track of the set of agents that are currently having the same request. Unfortunately, we have*

Proposition 3. *There exists an instance of system M such that under the set $\Psi = \{N_M^1, N_M^2\}$ of normative systems, both NC_1 and NC_2 are unsuccessful.*

This can be seen from a simple case where there are a single producer p_1 with $b_1 = 1$ and a set of consumers C such that $r_i = \{g_1\}$ for all $c_i \in C$. For the initial state, every consumer sends its request to p_1 , so $\{c_i \mid t_i = p_1\} = C$. For any path

ρ_1 of $K(N_M^1)$ and ρ_2 of $K(N_M^2)$, we have $\widehat{\rho}_1 = \widehat{\rho}_2 = s_0 s_2 s_1^1 s_2 \dots s_1^m s_2 s_1^v s_2 s_1^1 \dots$ where $s_0 = (1, (\emptyset, \perp, \perp), \dots, (\emptyset, \perp, \perp))$, $s_2 = (2, (\{g_1\}, g_1, p_1), \dots, (\{g_1\}, g_1, p_1))$, and s_1^i is different with s_2 in c_i 's local state, e.g., $s_1^1 = (1, (\{\}, \perp, \perp), \dots, (\{g_1\}, g_1, p_1))$. And therefore $O_{c_v}(\rho_1) = O_{c_v}(\rho_2) = \emptyset C(C \setminus \{c_1\})C(C \setminus \{c_2\})\dots$, i.e., the agent c_v 's observations are always the same⁴. That is, the new agent c_v finds that for ρ_1 , the single path on $K(N_M^1)$, there are $\rho_2 \in O_{c_v}^{-1}(O_{c_v}(\rho_1))$ and $\text{ind}(\rho_1) \neq \text{ind}(\rho_2)$. Therefore, neither NC_1 nor NC_2 can be successful in such a case.

The reasons for the above result may come from either the insufficient capabilities of the agent or the designing of normative systems. We explain this in the following example.

Example 6. First, consider that we increase the capabilities of the new agent by updating the rule R2b in Section 2.

R2b'. the new agent c_v may cancel its current request by letting $d_v = t_v = \perp$; all other consumer agents execute action a_{\perp} ; and let $k = 1$.

With this upgraded capabilities of the new agent, the NC_2 can be successful. The intuition is that, by canceling and re-requesting for at least twice, the ordering of consumer agents whose requests are satisfied can be different in two normative systems: with N_M^2 , there are other agents c_i between c_m and c_v , but with N_M^1 , their requests are always satisfied consecutively. Note that, by its new capabilities, c_v can always choose a producer agent which have more than 2 existing and future requests (Assuming that $n \ll m$, which is usual for a business ecosystem).

Proposition 4. With the new rule R2b', the NC_2 problem is successful on system M and the set $\Psi = \{N_M^1, N_M^2\}$.

However, the NC_1 problem is still unsuccessful, because the agent c_v may not move in such a smart way. For this, we replace N_M^1 with $N_M^6 = (Q^1, \delta_n^1, \delta_u^6, q_0^1)$ such that

⁴We reasonably assume that, for N_M^1 , when a producer sees c_v , it will adjust its range in normative states from $\{1, \dots, m\}$ to $\{1, \dots, m, v\}$.

- $\delta_u^6(q, s_2) = q$ and $\delta_u^6((y_1, \dots, y_n), s_1) = (y'_1, \dots, y'_n)$, s.t. $y'_j = ((y_j + 1) \bmod m) + 1$ for $j \in \{1, \dots, n\}$. Intuitively, the normative state increments by 2 (modulo m).

Proposition 5. *Both NC_1 and NC_2 problems are successful on system M and the set $\Psi = \{N_M^2, N_M^6\}$.*

7. The Complexity of Norm Recognition

The discussion in the last section clearly shows that, the two norm recognition problems are non-trivial. It is therefore useful to study if there exist efficient algorithms that can decide them automatically. In this section, we show a somewhat surprising result that the determination of NC_1 problem can be done in PTIME, while it is PSPACE-complete for NC_2 problem. Assume that the size of the set Ψ is measured over both the number of normative systems and the number of normative states. We have the following conclusions.

Theorem 2. *The NC_1 problem can be decided in PTIME, with respect to the sizes of the system and the set Ψ .*

Proof: First of all, by its definition in Definition 3, the unsuccessful answer to an NC_1 instance is equivalent to the existence of two infinite paths $\rho \in \text{Path}(K(N_0))$ and $\rho' \in O_x^{-1}(O_x(\rho))$ such that $\text{ind}(\rho') \neq \text{ind}(\rho)$. In the following, we give an algorithm to check such an existence.

Recall that x is the new agent. Let $K(N) = (S_N^\dagger, I_N^\dagger, T_N^\dagger, \pi_N^\dagger)$ be the Kripke structure obtained by applying the normative system $N \in \Psi$ on the system M . We define the function O_x^\dagger over the states $\bigcup_{N \in \Psi} S_N^\dagger$ by letting $O_x^\dagger((s, q)) = O_x(s)$ for all $(s, q) \in S_N^\dagger$. Moreover, we extend the ind function to work with the states in $\bigcup_{N \in \Psi} S_N^\dagger$ by letting $\text{ind}((s, q)) = \text{ind}(N)$ for q being a normative state of N .

A product system is constructed by synchronising the behaviour of two Kripke structures $K(N_0)$ and $K(N)$ for $N \in \Psi \setminus \{N_0\}$ such that the observations are always the same. Let $\text{Prop}' = \{\text{safe}\}$ be the set of atomic propositions. Formally, it is the structure $M' = (S', I', T', \pi')$ such that

- $S' = S_{N_0}^\dagger \times \bigcup_{N \in \Psi, N \neq N_0} S_N^\dagger$,

- $(s, t) \in I'$ if $s \in I_{N_0}^\dagger$ and $t \in \bigcup_{N \in \Psi, N \neq N_0} I_N^\dagger$ such that $O_x^\dagger(s) = O_x^\dagger(t)$.
- $((s, t), (s', t')) \in T'$ if and only if $(s, s') \in T_{N_0}^\dagger$, $(t, t') \in T_N^\dagger$ for some $N \in \Psi$ and $N \neq N_0$, and $O_x^\dagger(t) = O_x^\dagger(t')$ and
- $safe \in \pi((s, t))$ iff $ind(s) \neq ind(t)$.

Intuitively, the structure consists of two components: one component moves according to the Kripke structure $K(N_0)$ (that is, the currently active normative system), and the other moves according to other Kripke structures $K(N)$ by matching the observations of the new agent x . Therefore, we have the equivalence of the following two statements:

- the existence of two infinite paths $\rho \in Path(K(N_0))$ and $\rho' \in O_x^{-1}(O_x(\rho))$ such that $ind(\rho') \neq ind(\rho)$;
- the existence of an infinite path in the structure M' such that all states on the path are labelled with *safe*.

Then, the existence of an infinite path where all states are labelled with an atomic proposition can be reduced to 1) the removal of all states (and their related transitions) not labelled with the atomic proposition and then 2) the checking of reachable strongly connected components (SCCs).

For the complexity, we notice that M' is polynomial over M and Ψ , and the checking of reachable SCCs can be done in PTIME by the Tarjan's algorithm [17]. \square

Theorem 3. *The NC_2 problem is PSPACE-complete, with respect to the sizes of the system and the set Ψ .*

Proof:

We first show the upper bound: **PSPACE Membership**

The upper bound is obtained by having a nondeterministic algorithm which takes a polynomial size of space, i.e., it is in NPSpace=PSPACE.

First of all, by its definition in Definition 3, the successful answer to an NC_2 instance is equivalent to the existence of a finite paths $\rho \in Path(K(N_0))$ such that all

paths $\rho' \in O_x^{-1}(O_x(\rho))$ of the same observation belongs to $K(N_0)$, i.e., $ind(\rho') = ind(\rho)$. The idea of our algorithm is as follows. It starts by guessing a set of initial states of the structures $\{K(N) \mid N \in \Psi\}$ on which agent x has the same observation. It then continuously guesses the next set of states such that they are reachable in one step from some state in the current set and on which agent x has the same observation. If this guess can be done infinitely then the NC_2 problem is successful. This infinite number of guesses can be achieved with a finite number of guesses, by adapting the approach of LTL model checking [13].

Let $Prop'' = \{goal\}$ be the set of atomic propositions. We define the structure $M'' = (S'', I'', T'', \pi'')$ such that

- $S'' = S_{N_0}^\dagger \times \mathcal{P}(\bigcup_{N \in \Psi} S_N^\dagger)$,
- $(s, P) \in I''$ if $s \in I_{N_0}^\dagger$ and $P \subseteq \bigcup_{N \in \Psi} I_N^\dagger$ such that $t \in P$ iff $O_x^\dagger(s) = O_x^\dagger(t)$,
- $((s, P), (t, Q)) \in T''$ if and only if $(s, t) \in T_{N_0}^\dagger$ and $Q = \{t' \mid \exists s' \in P \exists N \in \Psi : (s', t') \in T_N^\dagger, O_x^\dagger(t') = O_x^\dagger(t)\}$, and
- $goal \in \pi((s, P))$ iff for all states $s \in P$ we have $ind(s) = ind(N_0)$.

Intuitively, each path of the structure M'' represents a path in $K(N_0)$ (in the first component) together with the set of paths with the same observation for agent x (in the second component). Therefore, we can have the equivalence of the following two statements:

- the existence of a finite paths $\rho \in Path(K(N_0))$ such that all paths $\rho' \in O_x^{-1}(O_x(\rho))$ of the same observation belongs to $K(N_0)$, i.e., $ind(\rho') = ind(\rho)$;
- in structure M'' , the existence of an initial state such that it can reach some state satisfying $goal$.

For the complexity of the algorithm, we note that although the system M'' is of exponential size, the reachability can be done on-the-fly by using a polynomial size of space.

We then show the lower bound: **PSPACE Hardness**

It is obtained by a reduction from the problem of deciding if, for a given nondeterministic finite state automaton A over an alphabet Σ , the language $L(A)$ is equivalent

to the universal language Σ^* . Let $A = (Q, q_0, \delta, F)$ be an NFA such that Q is a set of states, $q_0 \in Q$ is an initial state, $\delta : Q \times \Sigma \rightarrow \mathcal{P}(Q)$ is a transition function, and $F \subseteq Q$ is a set of final states. We construct a system $M(A)$ which consists of two subsystems, one of them simulates the behaviour of A and the other simulates the behaviour of the language Σ^* . The subsystems are reachable from an initial state s_0 by two actions a_1 and a_2 respectively. Let $\Sigma_1 = \Sigma \cup \{\perp\}$ such that $\perp \notin \Sigma$ is a symbol. Formally, we have that $M(A) = (S, \{Act_i\}_{i \in Agt}, \{L_i\}_{i \in Agt}, \{O_i\}_{i \in Agt}, I, T, \pi)$ is a single-agent system such that

- $Agt = \{x\}$,
- $S = S^1 \cup S^2 \cup \{s_0, s_{loop}\}$ with $S^1 = \Sigma_1 \times Q$ and $S^2 = \{s_a \mid a \in \Sigma_1\}$,
- $Act_x = \Sigma \cup \{a_1, a_2\}$,
- $L_x(s_0) = \{a_1, a_2\}$ and $L_x(s) = \Sigma$ for $s \in S \setminus \{s_0\}$,
- $O_x(s_0) = O_x(s_{loop}) = \perp$, $O_x((a, q)) = a$ for $(a, q) \in \Sigma_1 \times Q$, and $O_x(s_a) = a$ for $a \in \Sigma_1$, where $O = \Sigma_1$,
- $I(s_0) = 1$,
- the transition relation T consists of the following five sets of transitions:
 - $\{(s_0, a_1, (\perp, q_0)), (s_0, a_2, s_\perp)\}$; intuitively, from the initial state s_0 , it can transit into the subsystem $M_1(A)$ by taking action a_1 or the subsystem $M_2(A)$ by taking action a_2 ,
 - $\{((a, q), a_1, (a_1, q_1)) \mid q, q_1 \in Q, a \in \Sigma_1, a_1 \in \Sigma, q_1 \in \delta(q, a_1)\}$; intuitively, the subsystem $M_1(A)$ follows the behaviour of the automaton A ,
 - $\{((a, q), a_1, s_{loop}) \mid q \in Q, a, a_1 \in \Sigma, \delta(q, a_1) = \emptyset\}$; intuitively, in $M_1(A)$, all illegal actions take the system state into a designated state s_{loop} ,
 - $\{(s_{loop}, a, s_{loop}) \mid a \in \Sigma\}$; intuitively, the state s_{loop} is a loop state for all actions,
 - $\{s_a, a_1, s_{a_1} \mid a \in \Sigma_1, a_1 \in \Sigma\}$; intuitively, the subsystem $M_2(A)$ simulates the language Σ^* , and

- π will not be used.

The new agent x is the only agent of the system. On the system $M(A)$, we have two normative systems whose only difference is on the state s_0 : N_0 disallows action a_1 and N_1 disallows action a_2 . Formally, $N_0 = (\{t_0\}, \delta_n^0, \delta_u^0, t_0)$ such that

- $\delta_n^0(s_0, t_0) = \{a_1\}$, $\delta_n^0(s, t_0) = \emptyset$ for all $s \in S \setminus \{s_0\}$, and
- $\delta_u^0(t_0, s) = t_0$ for all $s \in S$.

and $N_1 = (\{t_1\}, \delta_n^1, \delta_u^1, t_1)$ such that

- $\delta_n^1(s_0, t_1) = \{a_2\}$, $\delta_n^1(s, t_1) = \emptyset$ for all $s \in S \setminus \{s_0\}$, and
- $\delta_u^1(t_1, s) = t_0$ for all $s \in S$.

Now we show that the universality of the NFA A is equivalent to the unsuccessful answer to the NC_2 problem on $M(A)$ and $\Psi = \{N_0, N_1\}$.

(\Rightarrow) Assume that the automaton A is universal. Then for all paths $\rho \in K(N_0)$, there exists another path $\rho' \in K(N_1)$ such that $O_x(\rho) = O_x(\rho')$. The inverse statement of the latter is that, there exists a finite path $\rho \in K(N_0)$ such that there exists no $\rho' \in K(N_1)$ such that $O_x(\rho) = O_x(\rho')$. The latter means that, all paths ρ' with $O_x(\rho) = O_x(\rho')$ are in $K(N_0)$, which is the statement of NC_2 problem.

(\Leftarrow) Assume that we have the unsuccessful answer to the NC_2 problem on $M(A)$ and $\Psi = \{N_0, N_1\}$. Then by the definition, it means that for all paths $\rho \in K(N_0)$, there exists another path $\rho' \in K(N_1)$ such that $O_x(\rho) = O_x(\rho')$. The latter is equivalent to the fact that the automaton A is universal.

□

8. Related Work

Normative multiagent systems have attracted many research interests in recent years, see e.g., [12, 18] for comprehensive reviews of the area. Here we can only review some closely related work.

Norm synthesis for static normative systems. As stated, most current formalisms of normative systems are static. [10] shows that this norm synthesis problem is NP-complete. [7] proposes a norm synthesis algorithm in declarative planning domains for reachability objectives, and [8] considers the on-line synthesis of norms. [19] considers the norm synthesis problem by conditioning over agents' preferences, expresses as pairs of LTL formula and utility, and a normative behaviour function.

Changes of normative system. [20] represents the norms as a set of atomic propositions and then employs a language to specify the update of norms. Although the updates are parameterised over actions, no considerations are taken to investigate, by either verification or norm synthesis, whether the normative system can be imposed to coordinate agents' behaviour to secure the objectives of the system.

Norm recognition. Norm recognition can be related to the norm learning problem, which employs various approaches, such as data mining [21] and sampling and parsing [22, 23], for the agent to learn social norms by observing other agents' behaviour. On the other hand, our norm recognition problems are based on formal verification, aiming to decide whether the agents are designed well so that they can recognise the current normative system from a set of possible ones. We also study the complexity of them.

Application of social norms Social norms are to regulate the behaviour of the stakeholders in a system, including sociotechnical system [24] which has both humans and computers. They are used to represent the commitments (by e.g., business contracts, etc) between humans and organisations. The dynamic norms of this paper can be useful to model more realistic scenarios in which commitments may be changed with the environmental changes.

9. Conclusions

In the paper, we first present a novel definition of normative systems, by arguing with an example that it can be a necessity to have multiple normative states. We study the complexity of two autonomy issues related to normative systems. The decidability (precisely, EXPTIME-complete) of norm synthesis is an encouraging result, suggesting that the maximum number of normative states is bounded for CTL objectives. For the

two norm recognition subproblems, one of them is, surprisingly, in PTIME and the other is PSPACE-complete. Because the first one suggests a better level of autonomy, to see if an agent can recognise the social norms, we can deploy a PTIME algorithm first. If it fails, we may apply a PSPACE algorithm to check the weaker autonomy.

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References

- [1] M. Fisher, L. Dennis, M. Webster, Verifying autonomous systems, *Communications of the ACM* 56 (9) (2013) 84–93.
- [2] Y. Shoham, M. Tennenholtz, On the synthesis of useful social laws for artificial agent societies, in: *AAAI 1992*, 1992, pp. 276–281.
- [3] W. van der Hoek, M. Roberts, M. Wooldridge, Social laws in alternating time: effectiveness, feasibility, and synthesis, *Synthese* 156 (1) (2007) 1–19.
- [4] T. Ågotnes, M. Wooldridge, Optimal social laws, in: *AAMAS 2010*, 2010, pp. 667–674.
- [5] M. Wooldridge, W. van der Hoek, On obligations and normative ability: Towards a logical analysis of the social contract, *Journal of Applied Logic* 3 (2005) 396–420.
- [6] T. Ågotnes, W. van der Hoek, M. Wooldridge, Normative system games, in: *AAMAS 2007*, 2007.
- [7] G. Christelis, M. Rovatsos, Automated norm synthesis in an agent-based planning environment, in: *AAMAS 2009*, 2009, pp. 161–168.

- [8] J. Morales, M. Lopez-Sanchez, J. A. Rodriguez-Aguilar, M. Wooldridge, W. Vasconcelos, Automated synthesis of normative systems, in: AAMAS 2013, 2013, pp. 483–490.
- [9] R. Fagin, J. Halpern, Y. Moses, M. Vardi, Reasoning About Knowledge, MIT Press, 1995.
- [10] Y. Shoham, M. Tennenholtz, On social laws for artificial agent societies: off-line design, Artificial Intelligence 73 (1-2) (1995) 231–252.
- [11] H. Chalupsky, Y. Gil, C. A. Knoblock, K. Lerman, J. Oh, D. V. Pynadath, T. A. Russ, M. Tambe, Electric elves: Applying agent technology to support human organizations, in: IAAI 2001, 2001, pp. 51–58.
- [12] G. Boella, L. van der Torre, H. Verhagen, Introduction to normative multiagent systems, Computational and Mathematical Organization Theory 12 (2-3) (2006) 71–79.
- [13] E. M. Clarke, O. Grumberg, D. Peled, Model Checking, The MIT Press, 1999.
- [14] O. Kupferman, M. Y. Vardi, Module checking, in: 8th International Conference on Computer Aided Verification (CAV1996), 1996, pp. 75–86.
- [15] M. Y. Vardi, P. Wolper, Automata-theoretic techniques for modal logics of programs, J. Comput. Syst. Sci. 32 (2) (1986) 183–221.
- [16] A. K. Chandra, D. C. Kozen, L. J. Stockmeyer, Alternation, Journal of the ACM 28 (1) (1980) 114–133.
- [17] R. E. Tarjan, Depth-first search and linear graph algorithms, SIAM Journal on Computing 1 (2) (1972) 146–160.
- [18] N. Criado, E. Argente, V. Botti, Open issues for normative multi-agent systems, AI Communications.
- [19] N. Bulling, M. Dastani, Verifying normative behaviour via normative mechanism design, in: IJCAI 2011, 2011, pp. 103–108.

- [20] M. Knobbout, M. Dastani, J.-J. C. Meyer, Reasoning about dynamic normative systems, in: JELIA 2014, 2014, pp. 628–636.
- [21] B. T. R. Savarimuthu, S. Cranefield, M. A. Purvis, M. K. Purvis, Identifying prohibition norms in agent societies, *Artificial intelligence and law*, 21 (1) (2013) 1–46.
- [22] N. Oren, F. Meneguzzi, Norm identification through plan recognition, in: COIN 2013@AAMAS, 2013.
- [23] S. Cranefield, T. Savarimuthu, F. Meneguzzi, N. Oren, A bayesian approach to norm identification (extended abstract), in: AAMAS 2015, 2015, pp. 1743–1744.
- [24] A. K. Chopra, M. P. Singh, From social machines to social protocols: Software engineering foundations for sociotechnical systems, in: WWW 2016, 2016, pp. 903–914.